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Failure of cord–rubber composites by pull-out or transverse fracture*

A. N. GENT
Institute of Polymer Science, The University of Akron, Akron, Ohio 44325, USA

G. S. FIELDING-RUSSELL, D. I. LIVINGSTON, D. W. NICHOLSON†
Research Division, The Goodyear Tire and Rubber Company, Akron, Ohio 44316, USA

A simple theoretical analysis has been developed for the force required to pull out an inextensible cord, or an array of cords, partly embedded in an elastic block. The analysis is based upon Griffith's fracture criterion: that energy supplied by the loading device as the cords are pulled out must be greater than the energy required to fracture the cord–block interface plus any increase in strain energy of the block itself. The pull-out force is obtained in this way as a function of cord diameter, the dimensions of the block, Young's modulus of the block material and the fracture energy per unit area of the interface. Measurements with brass-plated steel wire cords of various diameters, embedded to various depths in rubber blocks of varied dimensions, made of rubber having a wide range of Young's modulus, were all found to be in good agreement with the theoretical predictions. Moreover, the inferred value of the interfacial fracture energy is similar to a directly-measured value for rubber adhering to brass, about 20 kJ m^{-2}. The theoretical treatment also predicts that the total pull-out force for an array of n cords will increase in proportion to $n^{1/2}$, until transverse fracture intervenes. Both the proportionality to $n^{1/2}$ and the predicted transition to transverse fracture instead of cord pull-out have been observed. This broad agreement with the predictions of the theory suggests that the main factors governing cord pull-out have been taken into account.

1. Introduction

The fundamental problem in the fracture analysis of composites is to relate the breaking load to the dimensions of the assembly, its composition, and the properties of the components. Considerable success has been achieved along these lines by the use of a simple fracture criterion in terms of a characteristic energy requirement for failure. This energy criterion was first applied to the fracture of brittle solids by Griffith [1] and later to the separation of two adhering solids by Rivlin [2], Deryagin and Krotova [3], Lindley [4], Ripling *et al.* [5], Malyshev and Sagalnik [6], Williams [7], Gent and Kinloch [8], Bascom *et al.* [9] and others. Notable recent contributions by Kendall [10–12] form the basis for the present work.

In the analysis of a complex system, it is first necessary to recognize the mechanism of failure. Then an energy balance is formulated, in which changes in the strain energy of the stressed system and the potential energy of the loading device are equated to the energy requirements of the fracture process itself. This equation constitutes the basic criterion for fracture: an assembly will fail when, by so doing, enough mechanical energy is released to propagate the fracture.

Cord–rubber composites can fail in a variety of ways; by debonding of the components or by

*Contribution No. 626 from the Research Laboratories of the Goodyear Tire and Rubber Co.
†Present address: Naval Surface Weapons Center, Silver Spring, MD 20910, USA.

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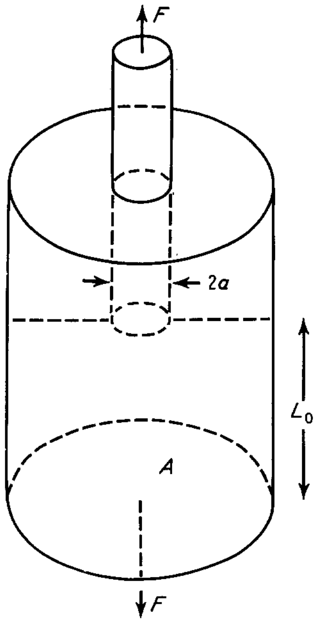

Figure 1 Forces applied to an embedded cord.

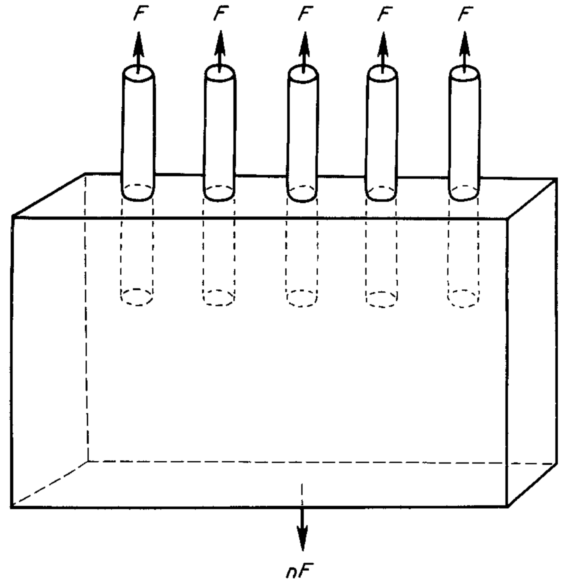

Figure 2 Pull-out of an array of cords.

fracture of either one. In this work one particularly important source of failure, associated with cord ends, is considered. If the level of adhesion between the rubber and the cord is relatively low, then debonding will start at the ends of the cords and propagate along them, resulting in cord "pull-out". If the level of adhesion is sufficiently high, then the ends of the cords, which act as sites of high stress concentration, may initiate transverse cracks. Either process leads to mechanical failure of the cord–rubber assembly.

A simple theoretical analysis of both modes of failure is developed in the following for two special cases: an isolated cord, and an array of cords embedded in a rubber block. It is assumed that the central region of the block is placed in a state of simple extension when the cords are subjected to tensile forces. This mechanical arrangement, shown schematically in Figs 1 to 3, is quite different from other cord pull-out geometries where the cord passes completely through the block, as in ASTM Test D 2229-73 [13], for example. No part of the rubber block is subjected to simple extension in such cases. Instead, the rubber is largely sheared between the central cord and an external clamp or holder, as discussed by Kendall [14]. The initial site and mode of propagation of a debond are not well defined in such cases. Failure initiates at either end of the cord and propagates inwards by cracking of the matrix or by a complex debonding process [14]. In contrast, failure in the present instance is always initiated at or near the cord ends and takes place either by cord pull-out or transverse fracture of the block. The corresponding theoretical analyses, developed below, are relatively simple and all of the parameters appearing in the theoretical relations can be measured independently so that there are no adjustable parameters.

Experimental measurements have been made on a variety of cord–rubber assemblies. They are compared with the theoretical predictions in later parts of the paper. The agreement obtained is sufficiently good to suggest that the theoretical analysis is basically correct. Attention is then drawn to a number of implications for the design of cord–rubber composites.

2. Theoretical considerations

2.1. General assumptions

In the following analysis the cords are assumed to be inextensible and the rubber block is assumed to follow a linear stress–strain relation in extension, with a slope (Young's modulus) denoted by E. Although these assumptions are somewhat more restrictive than is necessary, and they could well be relaxed in a more comprehensive analysis, they lead to particularly simple theoretical relationships, readily subjected to experimental test. Moreover, they are reasonably valid for rubber–cord composites where the cords are usually of steel, glass, or other high modulus fibres, and the breaking deformation of the composite is rather small. (It is,

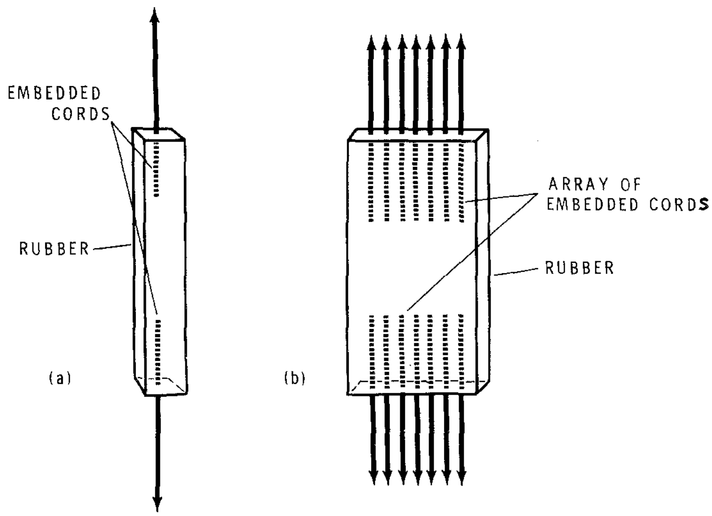

Figure 3 Test pieces used in pull-out experiments (a) single cord; (b) array of embedded cords.

of course, assumed in the analysis that the stresses set up in the cords are not large enough to break them.)

2.2. The isolated cord: conditions for cord pull-out or matrix fracture

An embedded cord of radius a is shown schematically in Fig. 1. The rubber block is shown as cylindrical in the figure, although its exact cross-sectional shape is immaterial. The essential assumption is that most of the block is in a state of simple extension under the action of the pull-out force F. If the cross-sectional area of the block is A, the corresponding tensile stress is

$$\sigma = F/A \tag{1}$$

and the strain energy stored in the rubber block is then

$$W = \sigma^2 A L_0/2E = F^2 L_0/2AE, \tag{2}$$

where L_0 is the initial length of the strained portion of the block.

Detachment of rubber from the end of the cord is assumed to take place quite easily because of the high stress concentration there. Thus the critical question is whether, under an increasing force F, an enclosed circular crack of radius a will grow transversely, resulting in fracture of the rubber matrix, or whether the cord will become debonded from the rubber progressively, starting at the cord end, until it is completely pulled out of the block.

The criterion for fracture of the block will be approximated by the Griffith solution for catastrophic growth of a small penny-shaped crack of initial radius a when a tensile stress σ_f is applied to the block [15]:

$$\sigma_f = (\pi E G_c/3a)^{1/2}, \tag{3}$$

where G_c denotes the fracture energy of the rubber, i.e., the energy required to tear through unit area, and Poisson's ratio has been taken to be 1/2. This relation assumes that strain energy is stored equally above and below the plane of fracture, whereas, in the present instance, rubber lying above the plane of the cord end will be constrained by adhesion to the cord and will not be strained as highly as rubber below the cord end. If the strain energy in the rubber above the plane of the cord end is regarded as negligibly small, then Equation 3 becomes

$$\sigma_f = (2\pi E G_c/3a)^{1/2}, \tag{4}$$

differing from the former expression by a factor of $\sqrt{2}$.

This relation was obtained by Mossakovskii and Rybka [16, 17] for the detachment of an elastic half-space from a rigid substrate when a circular debond of radius a is located at the interface. In practice, the fracture stress σ_f will probably lie between these two extreme values. The fracture force F_f can thus be represented by

$$F_f^2 = k\pi A^2 E G_c/3a, \tag{5}$$

where k is a numerical factor, lying between 1 and 2.

The criterion for detachment of rubber from the cord, leading to cord pull-out, may also be derived from a Griffith-type analysis. When a length c of the cord has become debonded, the volume of rubber subjected to simple extension is increased by an amount $(A - \pi a^2)c$. The total strain energy is correspondingly increased by $F^2c/2(A - \pi a^2)E$. However, the potential energy of the loading device is decreased by Fce, where e is the tensile strain in the detached portion of the block, given by $F/(A - \pi a^2)E$. The potential energy decrease is therefore exactly twice as large as the increase in total strain energy, so that a net loss of mechanical energy takes place, of amount $F^2c/2(A - \pi a^2)E$.

The energy expended in the debonding process may be expressed as $2\pi acG_a$, where G_a is the energy required to fracture unit area of the rubber–cord interface. Thus, debonding will take place when the mechanical energy released is sufficient to meet the requirements for debonding, i.e., when

$$F^2c/2(A - \pi a^2)E \geqslant 2\pi acG_a. \qquad (6)$$

Thus the pull-out force F_p is given by

$$F_p^2 = 4\pi(A - \pi a^2)aEG_a \qquad (7)$$

or, when the cross-sectional area of the cord can be regarded as small in comparison with that of the rubber block,

$$F_p^2 \simeq 4\pi AaEG_a. \qquad (8)$$

There are several interesting features of this result. First, the pull-out force is predicted to be independent of the embedded length of the cord and the overall length of the rubber block. Secondly, it is predicted to increase with the cross-sectional area A of the block in which the cord is embedded, and with the radius a of the cord. And, finally, it is predicted to increase with both Young's modulus E and detachment energy G_a, as in other Griffith-type analyses of fracture. These various predictions are compared with experimental results in a subsequent section.

The question of which type of failure will occur in practice is now considered: cord pull-out or transverse fracture of the rubber-block. Cord pull-out is predicted to occur when the pull-out force, given by Equation 8, is smaller than the force required for transverse fracture of the rubber block, given by Equation 5, i.e., when

$$k/12 > a^2G_a/AG_c. \qquad (9)$$

Thus, even when the interfacial bond strength, represented by G_a, is as high as the tear strength of the rubber, represented by G_c, the cord is predicted to pull out of the block if its radius a is less than $(kA/12)^{1/2}$, i.e., less than about one-half of the radius of the rubber block, or of an equivalent cylinder. Unfortunately, Equation 3 becomes invalid when the dimension of the initial circular crack, given by the cord radius, becomes comparable to the radius of the block. Thus the precise value of the maximum radius of the cord at which cord pull-out will take place instead of transverse fracture of the rubber block is somewhat uncertain. Nevertheless, it is clearly a relatively large dimension, comparable to the size of the block in which the cord is embedded, and cords having a diameter considerably smaller than the effective diameter of the rubber block are predicted to undergo pull-out under all circumstances.

2.3. An array of cords

An array of, for example, n cords, each of radius a, embedded in a rubber block of cross-sectional area A, is subjected to tensile forces tending to pull the cords out of the block (Fig. 2). The cords are shown arranged in a plane in the figure, but this is not an essential feature of the analysis. As before, it is assumed that detachment of rubber from the cord ends occurs quite easily, forming a series of circular cracks of radius a at the base of the cords. The condition for catastrophic transverse growth of any one of these cracks will be the same as before (Equation 3), where the critical tensile stress in the rubber block is given by

$$\sigma_f = nF_f/A, \qquad (10)$$

where F_f is the fracture force per cord. Thus Equation 5 becomes

$$F_f^2 = k\pi A^2EG_c/3an^2. \qquad (11)$$

A criterion for pull-out of the cords may be deduced in the same way as before. The net loss of mechanical energy in the system when debonding takes place along a length c of each cord becomes $n^2F^2c/2(A - n\pi a^2)E$, and the energy expended in debonding becomes $2\pi nacG_a$. Thus debonding will take place at a pull-out force F_p per cord given by

$$F_p^2 = 4\pi AaEG_a/n \qquad (12)$$

in place of Equation 8, when the total cross-sectional area of the cords is assumed to be small in comparison to the cross-sectional area A of the block.

Equation 11 predicts that the total applied force required to bring about transverse fracture, nF_f, will be the same for an array of cords as for a single cord. On the other hand, Equation 12 predicts that the total pull-out force nF_p will be larger for an array of n cords than for a single cord, by a factor $n^{1/2}$. As a result, the condition for pull-out is less easily met than before. The criterion for cord pull-out becomes

$$k/12n > a^2G_a/AG_c \qquad (13)$$

in place of Equation 9. If the bond strength G_a is equal to the tear strength G_c, and if n is taken as 10, for example, then Equation 13 predicts that the radius of each cord must be less than about one-seventh of the block radius (or equivalent radius) in order for the cords to be pulled out before the block fractures transversely. For a single cord, the critical radius was shown previously to be about one-half of the block radius. It is therefore clear that an array of cords is much less likely to pull out than a single cord, even though it has been assumed that the cords are far enough apart so that the local stress concentrations do not interact.

3. Experimental details

3.1. Test pieces

Two types of test-piece were used in the experiments. The first, which is similar to that shown in Fig. 1, contained two cords located axially (embedded in opposite ends of a rubber block of square cross-section, Fig. 3a [18]). The test-piece dimensions were 75 mm x 12 mm x 12 mm and each cord was embedded to a depth of 20 mm. The second test-piece, similar to that shown in Fig. 2, contained two arrays of cords embedded in opposite faces of a rubber block of rectangular cross-section (Fig. 3b). The test-piece dimensions were 75 mm x 36 mm x 5 mm and the arrays of cords were embedded to a depth of 20 mm. The number of cords in each array was varied from 1 to 13.

Failure was induced in each test-piece by gripping the opposing free cord ends in the clamps of a tensile testing apparatus and pulling them apart until one of the cords or arrays of cords pulled out of the block or until the block fractured transversely. The former type of failure is termed here "adhesive" failure, because the emerging cords seemed to be clean and free from adhering rubber, from visual inspection. The latter type of failure, when a crack started at the embedded cord ends and ran at right angles to the cord direction is termed here "cohesive" failure. In each case, the maximum tensile force required to bring about failure was recorded.

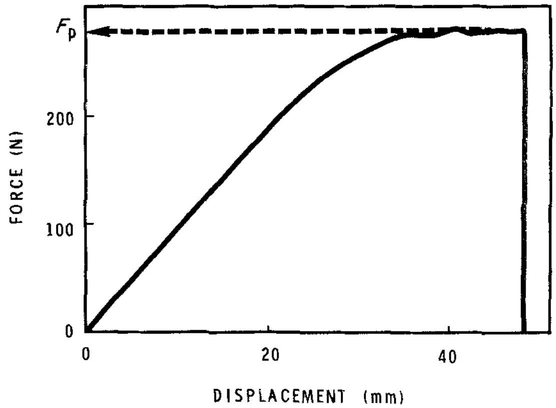

Figure 4 Typical force displacement relation for cord pull-out.

A typical pull-out force–displacement relation is shown in Fig. 4. Values of Young's modulus for the rubber were determined from the slope of the initial linear portion or from a separate indentation measurement on the rubber block [19].

3.2. Materials

Rubber blocks were made from *cis*-polyisoprene with various amounts of carbon black incorporated, and cross-linked by sulphur during a hot-moulding process. The cords were placed in the mould initially and the rubber block was formed and cross-linked in contact with them, so that it adhered to the cords quite strongly. By varying the amount of carbon black in the rubber compound and the degree of cross-linking, the Young's modulus of the blocks was varied from 1.7 to 16 MPa.

The cords consisted of brass-plated steel wire cords, as used in tyre manufacture. They were made by twisting between 5 and 29 monofilament wires together, each wire having a diameter of about 0.2 mm. The perimeter of the cord cross-section, shown schematically in Fig. 5, was calculated for each cord from the filament diameters, assuming close packing.

4. Experimental results and discussion

4.1. Pull-out forces for a single cord

As all of the test pieces were made from a single elastomer, cross-linked with sulphur in contact with the same substrate material, namely, brass-coated steel, the fracture energy G_a for the rubber–cord interface is likely to be similar in magnitude in all cases. Equation 3 then predicts that the pull-out force F_p for a given cord will increase with Young's modulus E of the rubber

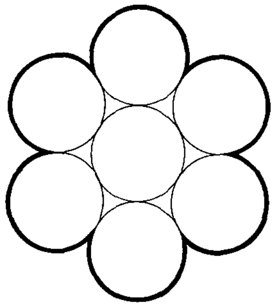

Figure 5 Sketch showing cord perimeter.

block, in proportion to $E^{1/2}$. This prediction was found to be correct, as shown in Fig. 6, where results are plotted for a wide range of values of E. Moreover, when the cross-sectional area of the rubber block was varied by a factor of about 4, the pull-out forces increased by a factor of about 2, in accordance with the predicted dependence upon the square root of the cross-sectional area A (Equation 8). When two different cords were used, having effective radii in the ratio 1.7:1, the pull-out forces were found to be in the ratio 1.4:1 (Fig. 7), in reasonable agreement with the square-root dependence upon cord radius a predicted by Equation 8.

When the cross-sectional dimensions of the rubber block were increased further, a limiting condition was observed when the pull-out force no longer continued to increase in proportion to the square root of the cross-sectional area but rather became independent of A (Fig. 8). Apparently, when the cross-sectional dimensions of the block become relatively large, the rubber between the cord ends can no longer be regarded as being in a state of simple extension under the action of the pull-out force. When this basic premise of the theory no longer holds, then a dependence of the pull-out force upon the square root of the cross-sectional area of the block is no longer observed.

Equation 8 predicts that the pull-out force F_p should be independent of the embedment depth of

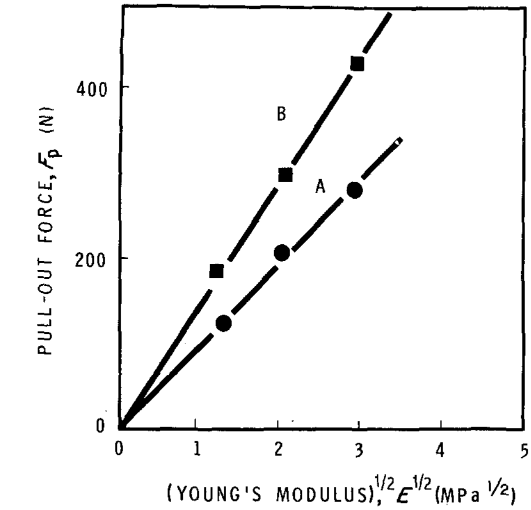

Figure 7 Pull-out force against square root of rubber modulus for two values of the effective cord radius: A, 0.44 mm; B, 0.76 mm. Cross-sectional area of block = 144 mm².

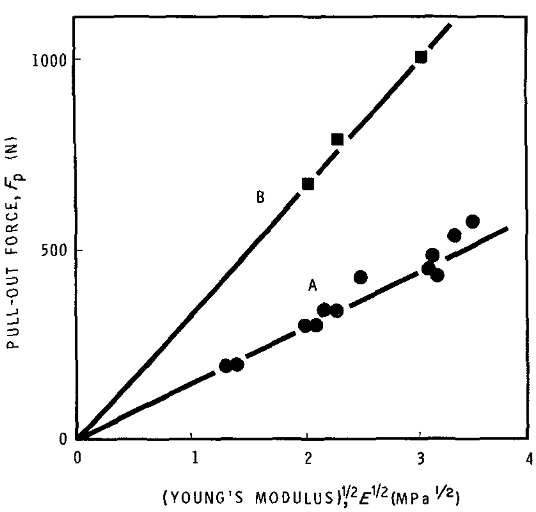

Figure 6 Pull-out force against square root of rubber modulus for two values of the test-piece cross-sectional area. A, 144 mm²; B, 625 mm². Effective cord radius = 1.2 mm.

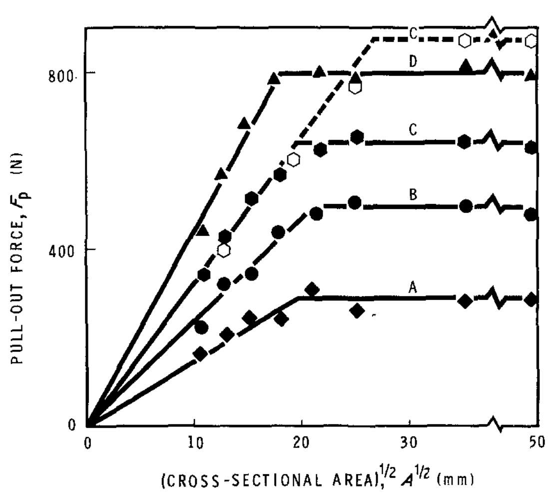

Figure 8 Pull-out force against square root of test-piece cross-sectional area for various values of rubber modulus: A, 1.69 MPa; B, 4.00 MPa; C, 8.35 MPa; D, 12.25 MPa. Broken line: results obtained with rubber blocks of square cross-section.

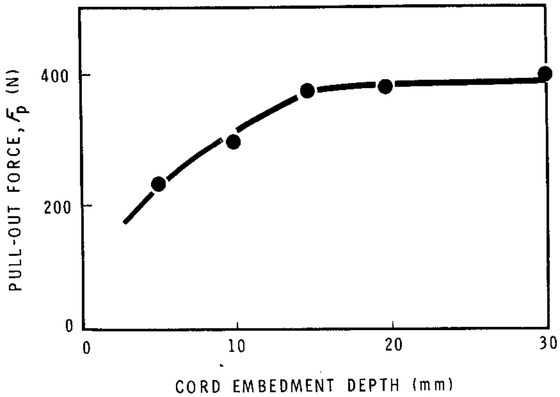

Figure 9 Pull-out force against cord embedment depth ($E = 8.35$ MPa and $a = 0.76$ mm).

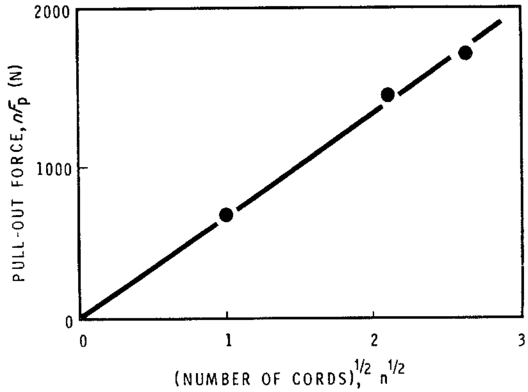

Figure 10 Pull-out force against square root of number n of cords embedded ($E = 8.35$ MPa and $a = 1.1$ mm).

the cord. Although this was found to be the case for embedment depths of 15 mm or more, lower pull-out forces were recorded when the cord was embedded for smaller distances than this (Fig. 9). It seems probable that the cord embedment depth should be considerably greater than the effective diameter of the rubber block for the shear deformation of the rubber surrounding the cord to be negligibly small, as is assumed in the theoretical treatment.

Thus, except for cords that were not embedded to a depth greater than the effective diameter of the rubber block or for specimens with a relatively large cross-sectional area, having dimensions comparable to the separation between the cord ends, the pull-out force has been found to vary with the cord radius a, the block cross-sectional area A and Young's modulus of the rubber E in good agreement with the predictions of Equation 8.

From the experimental data presented in Figs 6 to 8, the mean value of the adhesion fracture energy G_a was calculated by means of Equation 8 to be $17 \pm 3\,\text{kJ m}^{-2}$. This value is in good agreement with independent measurements of G_a for the same rubber–substrate combination obtained from a peeling experiment, namely $20\,\text{kJ m}^{-2}$ [18]. Thus the mechanics of cord pull-out appear to be well described by the present theory.

Possible contributions to the pull-out force arising from friction between the detached cord and the surrounding rubber have been ignored in the analysis. Values of the pull-out force for unbonded cords were found to be quite small, less than one-tenth of those for corresponding bonded cords, and it therefore seems appropriate to neglect them. However, the frictional contribution is expected to increase with the cord embedment depth [20, 21]. It is therefore likely to become significant, and eventually dominant, as the depth of embedment increases beyond the range employed here.

4.2. Pull-out forces for an array of cords and the cohesive–adhesive transition

Using one particular rubber compound $E = 8.35$ MPa, and one particular cord, having an effective radius $a = 1.1$ mm, the pull-out force F_p or transverse fracture force F_f was determined as a function of the number of cords n embedded in a parallel array (Fig. 3b). As predicted by Equation 12, the total pull-out force nF_p was found to increase with n in proportion to $n^{1/2}$ (Fig. 10) up to a value of n of 7. Above this value, transverse fracture of the rubber block took place, initiated by the cord ends, at a total force nF_f that was found to be independent of n (Table I) in agreement with Equation 11. Thus the way in which the pull-out or fracture forces for an array of cords depended upon the number of cords in the array is in good agreement with the theoretical predictions.

The critical number n_c of cords above which transverse fracture will occur and below which cord pull-out will occur, can be caluclated from Equation 13. For this purpose, the value of the

TABLE I Failure force and mode of failure as a function of the number of cords embedded

Number of cords	Failure force (N)	Failure mode
1	650	Pull-out of cords
5	1425	Pull-out of cords
7	1715	Pull-out of cords
10	1570	Transverse fracture of rubber
13	1550	Transverse fracture of rubber

adhesion fracture energy G_a is assumed to be equal to the cohesive fracture energy G_c of the rubber matrix, in view of the strong bond obtained between rubber and brass-plated steel. The measured value of G_a, about $20\,\text{kJ m}^{-2}$, is, in fact, not much smaller than the tear energy of the rubber G_c, about $40\,\text{kJ m}^{-2}$. The numerical factor k was given a value of unity, and appropriate values were assigned to a and A of 1.1 mm and $180\,\text{mm}^2$, respectively. Equation 13 then yields a critical value n_c of about 12 cords, in reasonable agreement with the observed value of 7–10 cords. Thus the observed transition to transverse fracture instead of cord pull-out, as the number of cords was increased, is predicted satisfactorily.

Instances of transverse fracture of the rubber block were also noted when single cords of relatively large diameter were used, especially with rather weak rubber compounds. These observations are qualitatively in accord with the theoretically predicted conditions for a transition from cord pull-out to transverse fracture (Equation 9), but a quantitative comparison has not yet been made.

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